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# METHODOLOGY FOR DETERMINING EXPERIMENTAL PLOT SIZE FOR INTERCROPPING EXPERIMENTS

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SUMMARY

Uniformity trials with monocrops are legion but with intercropping systems are seldom reported in literature. The methodology for arriving at the optlmum plot size for intercropping systems should suit the method of analysis. Two methods in vogue are : (1) comparing taeatments through an index such as LER, which is a linear function of the yields of the two species and (2) bivariate analysis advocated by Pearce and several others. Therefore, the methodology suited to both chese procedu:es has been worked out in this paper and exemplified with the help of data generated by the authors by conducting uniformity trial with sorghum + pigeonpea intercropping system for two seasons.

*Keywords* : Intercropping, Land Equivalent Ratio, Bivariate analysis, Smith's empirical law, Convenient plot size.

## Introduction

Analysis of intercropping experiments involves more complications compared to sole cropping. Two crop intercropping experiments are most common, particularly in dryland agriculture (Ramanatha Chetty, [6]).

The popular method in analysing these experiments is as a univariate problem by converting the bivariate situation to univariate one such as the total monetary values or some function of the combined yields ('indices') which characterise the competition between the crops. Among such indices, Land Equivalent Ratio (LER) is widely used and the details about asessing the yield advantages through LER are discussed by Mead and Willey [2]. Since the problem is bivariate, Pearce and Gilliver [5] suggested bivariate analysis which is a two variate special case of the standard multivariate analysis. This paper is concerned in developing appropriate methodology for suggesting plot sizes for precise comparison of treatment means based on (i) bivariate analysis and (ii) LER analysis. The results of two years' uniformity trials on intercropping conducted by the authors are used for exemplifying the methodology.

## 2. Bivariate Variance Law for Different Plot Sizes

In the case of multivariate Gauss-Morkoff set up (Rao, [8])

$$\Lambda = \frac{|R_0|}{|R_1|} \text{ with parameters } (p, t - q, q) \tag{1}$$

is distributed as the product of independent beta variables under the assumption of normality with parameters,

$$\frac{(t - q - p + 1}{2}, \frac{q\ )}{2} \ldots \frac{(t - q}{2}, \frac{q\ )}{2}.$$

Here

$p$ is the number of variables,
$t$ is total d.f.,
$q$ is d.f. due to deviation from hypothesis,
$R_0 = (R_0\,(i, j))$ is the matrix of residual sum $(p \times p)$ of squares and products,
$R_1 = (R_1\,(i,j))$ is the matrix of (residual + $(p \times p)$ deviation from hypothesis) sum of squares and products,
$|R_0|$ and $|R_1|$ are the determinants of $R_0$ and $R_1$.

The above statistic is used to test the deviation from hypothesis when there are $p$ ($\geqslant$ 2) variables. In the bivariate case i.e., when $p = 2$, the ratio

$$\frac{1 - \sqrt{\Lambda}}{\sqrt{\Lambda}} \frac{(t - q - 1)}{q} \tag{2}$$

follows $F$-distribution with 2 $q$ and 2 $(t-q-1)$ d.f. (Wilks, [10]) after a transformation. The same method is used in a different but equivalent way by Pearce and Gilliver to test the significance of the treatment means in intercropping experiments.

From (1) and (2) it is clear that a smaller value of $\sqrt{\Lambda}$ will result in the greater precision of testing the treatment means. Since the total variability remains the same $|R_1|$ is constant. Hence the criteria in selecting the experimental plot should be such that the value of $\sqrt{|R_0|}$ should be small. For this, we need the information of $R_0$ for different plot sizes. The direct way of obtaining this information is from uniformity trials by growing two crops in recommended geometry under uniform treatment, in an intercropping system.

In a uniformity trial with intercropping of two crops, the observed yields follow the bivariate model.

$$\underline{Y}_{xk} = \underline{\mu}_{xk} + \underline{\varepsilon}_{xk} \quad k = 1, 2, \ldots, n_x; \tag{3}$$

where

$$\underline{Y}'_{xk} = (Y_{xk1}, Y_{xk2}),$$
$$\underline{\mu}'_{xk} = (\mu_{xk1}, \mu_{xk2}), \; \underline{\varepsilon}'_{xk} = (\varepsilon_{xk1}, \varepsilon_{xk2}),$$

$Y_{xk1}$ and $Y_{xk2}$ are the observed yields of crop 1 and crop 2 from $k$th plot of size $x$ basic units,

$\mu_{xk1}$ and $\mu_{xk2}$ are the expected yields of crop 1 and crop 2 from $k$th plot of size $x$ basic units, and

$\varepsilon_{xk1}$ and $\varepsilon_{xk2}$ are errors of crop 1 and crop 2 from $k$th plot of size $x$ basic units with the assumptions :

$$E(\varepsilon_{xk1}) = 0, \; E(\varepsilon_{xk2}) = 0,$$
$$\text{Cov}(\varepsilon_{xk1}, \varepsilon_{xk2}) = \sigma_{12x} \tag{4}$$
$$V(\varepsilon_{xk1}) = \sigma_{11x} \text{ and } V(\varepsilon_{xk2}) = \sigma_{22x}$$

and they are independent for different plots.

$n_x$ is the nnmber of plots of size $x$ basic units. Let $R_{x0}$ be the corresponding $R_0$ for plot size of $x$ basic units.

From the model (3) and the assumptions (4)

$$\hat{\sigma}_{ijx} = \frac{R_{x0}(i, j)}{(n_x - 1)} = V_{(x)ij}, \text{ say; } i, j = 1, 2.$$

Hence, minimizing $\sqrt{|R_{x0}|}$, is same as minimizing

$$U_x = \{V_{(x)11} V_{(x)22} - V^2_{(x)12}\}^{1/2} \tag{5}$$

In the uniariate case, Smith (1938) proposed an empirical law as

$$V_x = V_1/X^b \tag{6}$$

where

$V_x$ is the variance of yield per unit area among plots of $x$ basic units,

$V_1$ is the varianse among plots of size unity, and

$b$ is the heterogeneity coefficient.

Let $W_x$ be the value of $U_x$ per unit area among plots of size $x$ basic units. Then

$$W_x = \frac{U_x}{x^2} = \left\{\frac{V_{(x)11}\, V_{(x)22}}{x^4} - \frac{(V_{(x)12})^2}{x^4}\right\}^{1/2}$$

$$= \{V_{x11}\, V_{x22} - V^2_{x12}\}^{1/2} \tag{7}$$

where

$$V_{xij} = \frac{V_{(x)ij}}{x^2} \qquad i, j = 1, 2$$

From (6) we have

$$V_{xij} = \frac{V_{ij}}{X^{b_{ij}}} \tag{8}$$

From (7) and (8) it follows that

$$W_x^2 = \frac{V_{11}\, V_{22}}{X^{b11+b22}} \left[1 - \rho^2 \; \frac{X^{b11+b22}]}{X^{2b12}}\right.$$

i.e.

$$W_x = \sqrt{\frac{V_{11}\ V_{22}}{X^{b11+b22}}} \left[1 - \frac{\rho^2\, X^{b11+b22}}{X^{2b12}}\right]^{1/2}$$

$$= \frac{W_1}{x^g}, \text{ say} \tag{9}$$

where $\rho$ is the correlation coefficient between crop 1 and crop 2 yields among the plots of size unity. When $\rho$ is zero then $g$ is the arithmetic mean of the individual crop heterogeneity coefficients $b_{11}$ and $b_{22}$. When $\rho \neq 0$, then the $g$ is a function of $b_{11}$, $b_{22}$ and also $b_{12}$. The heterogeneity coefficient $b_{12}$ will adjust $g$ to take care of the covariance between the crops. $g$ is the index of soil heterogeneity in the care of intercropping (analogous to '$b$' in monocrop experiments).

## 3. Variance Law in LER

As mentioned earlier, transformation of the yields into LER appears to be most satisfactory since LER values are based on sound agronomic meaning. Since LER is sum of two ratios, there are some theoretical problems in analysing them. But Oyejola and Mead (1982) showed empirically that the deviations from the analysis of variance are not serious when sole crop yields are properly selected in the calculation of LER. For calculating the LER values on micro-plot basis, pure stand yields of the two crops grown in an adjacent area as monocrops are utilized. The degree and distribution of the variability in LER values are studied by fitting Smith's law :

$$V_x \text{ (LER)} = \frac{V_1 \text{ (LER)}}{x^b} \tag{10}$$

where $V_x$ (LER) is the variance per plot of LER values among the plot of size $x$ basic units, $V_1$ (LER) is the value of $V_x$ (LER) when $x$ is the unity and $b$ is the heterogeneity coefficient.

## 4. Optimum Plot Size

To select the optimum plot size the convenient plot size method suggested by Hatheway [1] was adopted. The investigator often wishes to know the number of replications and the size of the plot required to detect a difference of a specified magnitude between two treatment means, irrespective of cost. The relationship between plot size, number of replications and the difference to be detected is as follows :

$$d^2 = \frac{2 (t_1 + t_2)^2 \; C_1^2}{r x^b} \tag{11}$$

where $d$ is the true difference to be detected between the two treatmental means expressed as percentage, $t_1$ is the tabulated value of $t$ in the test of significance, $t_2$ is the tabulated value of $t$ corresponding to $2(1 - p)$, where $p$ is the probability of obtaining a significant result, $b$ is the heterogeneity coefficient, $r$ the number of replications and $C_1$ the coefficient of variation due to basic size plots. Plotting $d$ value against the plot size $x$ by varying $r$ will help in arriving at a plot size for a required '$d$' value.

## 5. Results of Sorghum + Pigeonpea Intercropping Uniformity Trial

The authors conducted uniformity trial on sorghum intercropped with pigeonpea in 2 : 1 row ratio in two successive rainy seasons of 1981 and 1982 at Hayatnagar Research Farm, 15 km. from Hyderabad, under dryland conditions. The row width was 45 cm. The yields of crops were recorded for units of plots of basic size of 1.82 $m^2$ i.e. 1.35 $m$ × 1.35 $m$.

The suggested bivariate variance law and the variance law (LER) have been fitted for two years data of uniformity trials on sorghum and pigeonpea intercropping conducted by the authors during 1981 and 1982.

The values of the estimates of the correlation coefficient ($\rho_x$) and $W_x$ are given in Table 1 for all the possible simulated plot sizes for each of the two years. The $\rho_x$ values are significant and this suggests the need for bivariate analysis.

The fit of the suggested equations is quite satisfactory in both years. In the ease of individual crop yields the $b$ values for sorghum and pigeonpea respectively are 0.3415 and 0.2445 during 1981; 0.5343 and 0.2160 during 1982 for sorghum and pigeonpea respectively. If the crops are independent, then the $g$ coefficient is just the arithmetic mean of the $b$ coefficients corresponding to the two crops. In the present situation the estimated $g$ coefficients are larger than the expected $g$ coefficient when the crops are independent. This is because of the significant covariance between the two crops. So the covariance between the crops increases the degree of intraclass correlation and this suggests the need for larger plot size for testing the treatment effects, more precisely. In the univariate case the alternative way of estimating the $b$ values are by

$$CV_x = \frac{CV_1}{x^{b/2}}$$

where $CV_x$ is the coefficient of variation among the plots of size $x$ basic units.

The same type of relation holds in bivariate situation. Here

$$CV_x\,(\text{biv}) = \left[\frac{U_x}{(\bar{Y}_{x1}\ \bar{Y}_{x2})}\right]^{1/2}$$

where $\bar{Y}_{x1}$ and $\bar{Y}_{x2}$ are the mean yields of crop 1 and crop 2 of plots of size with $x$ basic units. The above definition of $CV_x$ (biv) i.e. coefficient

TABLE 1—VALUES OF $W_x$ AND $\rho_x$ FOR DIFFERENT PLOT SIZES AND SHAPES FOR THE TWO SEASONS

| *Sl. No.* | *Plot size/ shape* | $W_x$ | | $\rho_x$ | |
|---|---|---|---|---|---|
| | | *1981* | *1982* | *1981* | *1982* |
| 1 | 1 × 1 | 0.00584 | 0.00386 | −0.09 | −0.10 |
| 2 | 1 × 2 | 0.00421 | 0.00248 | −0.09 | −0.16 |
| 3 | 1 × 3 | 0.00344 | 0.00238 | −0.18 | −0.21 |
| 4 | 1 × 4 | 0.00305 | 0.00213 | −0.18 | −0.22 |
| 5 | 1 × 6 | 0.00243 | 0.00182 | −0.25 | −0.27 |
| 6 | 1 × 8 | 0.00197 | 0.00145 | −0.33 | −0.29 |
| 7 | 2 × 1 | 0.00435 | 0.00267 | −0.11 | −0.17 |
| 8 | 2 × 2 | 0.00328 | 0.00208 | −0.12 | −0.22 |
| 9 | 2 × 3 | 0.00286 | 0.00183 | −0.22 | −0.26 |
| 10 | 2 × 4 | 0.00254 | 0.00167 | −0-22 | −0.26 |
| 11 | 2 × 6 | 0.00209 | 0.00143 | −0.30 | −0.32 |
| 12 | 2 × 8 | 0.00166 | 0.00113 | −0.37 | −0.32 |
| 13 | 3 × 1 | 0.00391 | 0.00228 | −0.14 | −0.21 |
| 14 | 3 × 2 | 0.00307 | 0.00185 | −0.14 | −0.25 |
| 15 | 3 × 3 | 0.00274 | 0.00169 | −0.23 | −0.29 |
| 16 | 3 × 4 | 0.00242 | 0.00153 | −0.23 | −0.29 |
| 17 | 3 × 6 | 0.00205 | 0.00132 | −0.30 | −0.35 |
| 18 | 3 × 8 | 0.00160 | 0.00103 | −0.40 | −0.36 |
| 19 | 4 × 1 | 0.00353 | 0.00202 | −0.11 | −0.20 |
| 20 | 4 × 2 | 0.00284 | 0.00163 | −0.14 | −0.20 |
| 21 | 4 × 3 | 0.00258 | 0.00148 | −0.23 | −0.29 |
| 22 | 4 × 4 | 0.00225 | 0.00138 | −0.24 | −0.29 |
| 23 | 4 × 6 | 0.00192 | 0.00115 | −0.32 | −0.37 |
| 24 | 4 × 8 | 0.00149 | 0.00093 | −0.41 | −0.36 |
| 25 | 6 × 1 | 0.00319 | 0.00157 | −0.12 | −0.31 |
| 26 | 6 × 2 | 0.00261 | 0.00128 | −0.13 | −0.37 |
| 27 | 6 × 3 | 0.00242 | 0.00121 | −0.19 | −0.39 |
| 28 | 6 × 4 | 0.00212 | 0.00108 | −0.23 | −0.41 |
| 29 | 6 × 6 | 0.00188 | 0.00092 | −0.30 | −0.47 |
| 30 | 6 × 8 | 0.00148 | 0.00078 | −0.38 | −0.45 |
| 31 | 8 × 1 | 0.00294 | 0.00124 | −0.12 | −0.28 |
| 32 | 8 × 2 | 0.00242 | 0.00099 | −0.15 | −0.35 |
| 33 | 8 × 3 | 0.00228 | 0.00095 | −0.22 | −0.37 |
| 34 | 8 × 4 | 0.00197 | 0.00085 | −0.24 | −0.41 |
| 35 | 8 × 6 | 0.00173 | 0.00068 | −0.34 | −0.51 |
| 36 | 8 × 8 | 0.00132 | 0.00061 | −0.43 | −0.50 |

The fitted equations are

$$W_x = 0.005183\ X^{-0.3130} \qquad R^2 = 0.88\ (1981)$$

$$W_x = 0.003705\ X^{-0.4074} \qquad R^2 = 0.95\ (1982)$$

The $g$ coefficients are highly significant.

of variation of plots of $x$ basic units in bivariate case leads to

$$CV_x\,(\text{biv}) = \frac{CV_1\,(\text{biv})}{x^{g/2}} \qquad (12)$$

$$\text{Since } CV^2_x\,(\text{biv}) = \frac{U_x}{\bar{Y}_{x1}\,\bar{Y}_{x2}} = \frac{U_x}{x^2\,\bar{Y}_1\,\bar{Y}_2}$$

$$= \frac{1}{C}\,\frac{(U_x)}{(x^2)} = \frac{W_x}{C}$$

where $C = \bar{Y}_1 \cdot \bar{Y}_2$ is a constant.

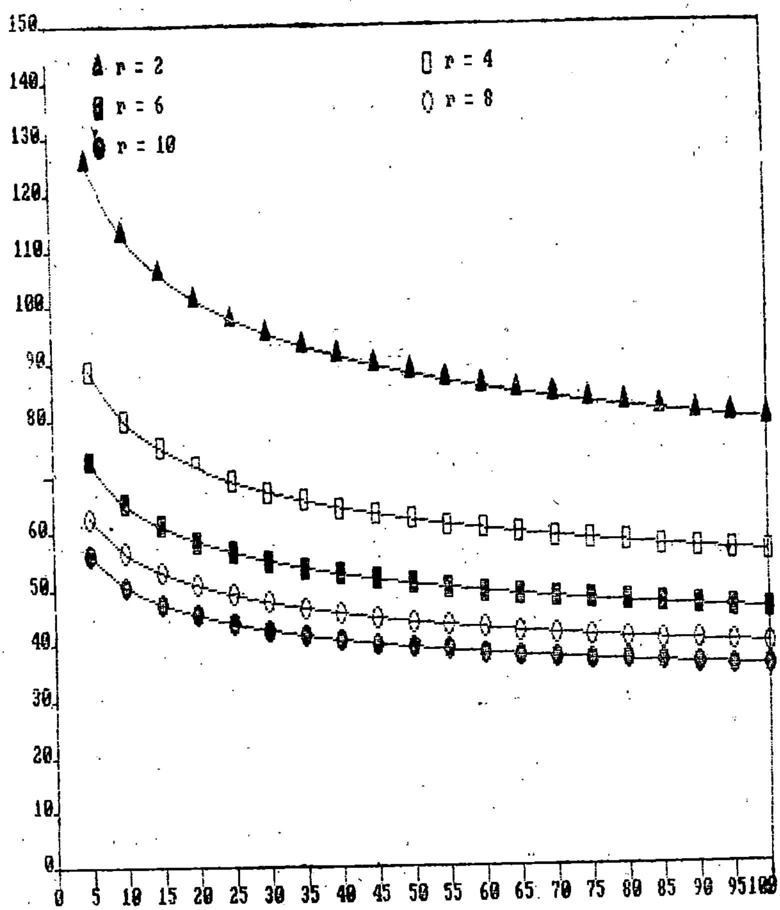


Fig. 1 (a)

*Note* : Figs. 1(a), 1(b), 2(a) and 2(b) depict the relationship between plot size $(X)$ and number of replications and true difference to be detected between any two treatments as significant, expressed as percentage $(d)$ in sorghum + pigeonpea intercropping system, during 1981 and 1982 based on (i) bivariate and (ii) LER analysis.

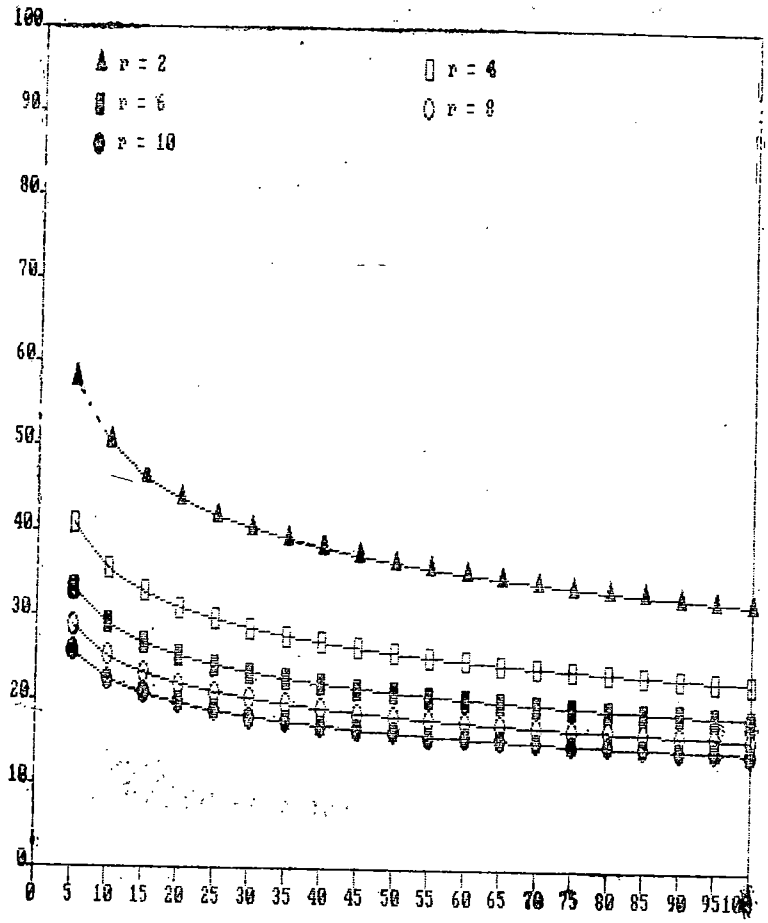


Fig. 1 (b)

In the case of LER as the variate, we have the following fitted models :

$$V_x = 0.11059/X^{-0.3794} \qquad R^2 = 0.85 \text{ (1981)}$$

$$V_x = 3.00184/X^{-0.3984} \qquad R^2 = 0.91 \text{ (1982)}$$

The curves to find the convenient plot size are drawn based on the equation (11) for the number of replications 2, 4, 6, 8 and 10. Figures 1 (a) and 1 (b) represent the bivariate case and Figures 2 (a) and 2 (b) correspond to LER, for the two seasons, respectively.

Here, $CV_1$ (biv) = 53.67 (1981)

= 43.63 (1982)

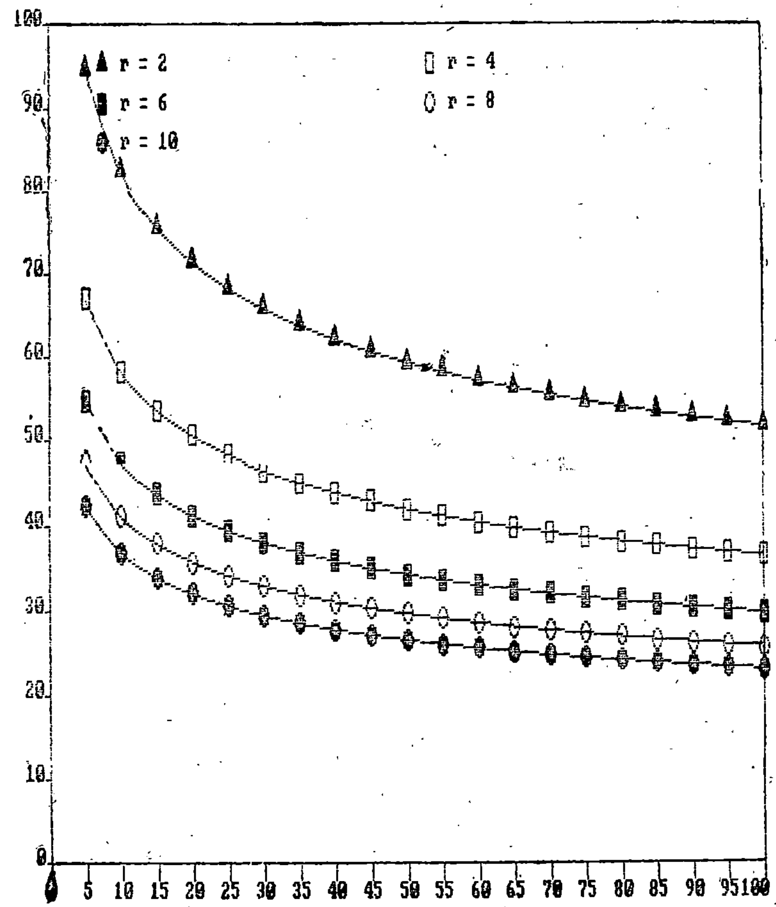


Fig. 2 (a)

$$CV_1\,(\text{LER}) = 35.39 \;(1981)$$
$$= 26.41 \;(1982)$$

The values $t_1$ and $t_2$ are 2.145 and 0.868 respectively, at $p = 0.05$.

## 6. Conclusion

In this paper, basically Smith's empirical variance model has been extended to bivariate situation. The extension to situations of more than two variables is straightforward and it does not involve any additional complications. The suggested model can be used not only in intercropping studies but also in the situations where $p$ characters such as yield, number of plants, etc., are recorded from the same plot and are to be

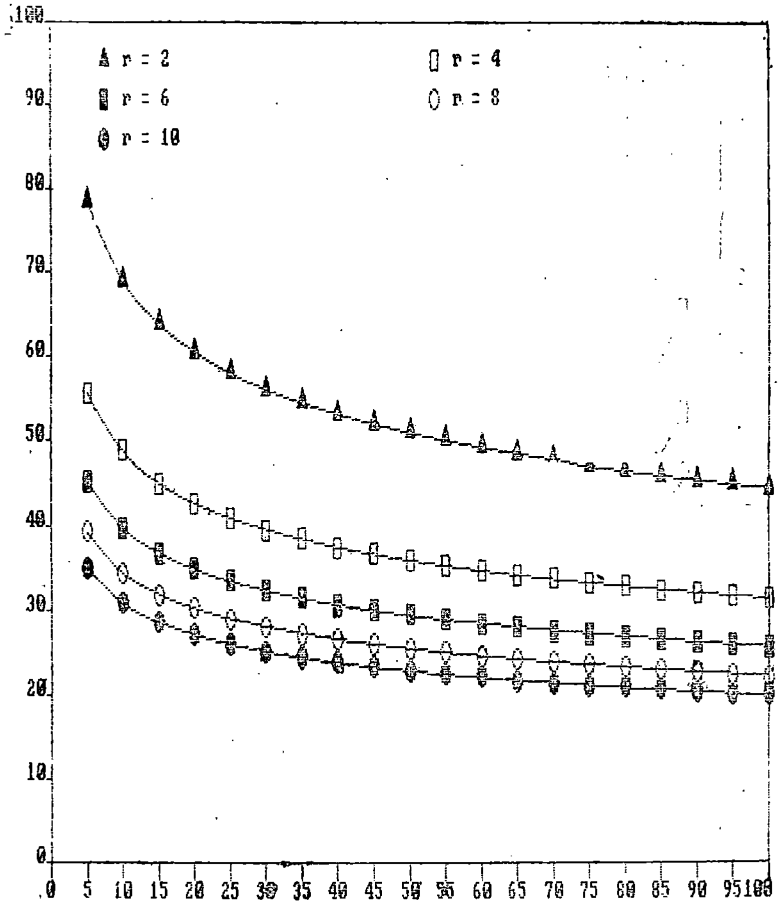


Fig. 2 (b)

analysed by multivariate methods. Here, the method of incorporating the effect of plot shape has not been answered but it appears that the univariate methods discussed by Narayana Reddy and Ramanatha Chetty [3] and Ramanatha Chetty and Narayana Reddy [7] hold good. The other aspects of considerable importance are to study the effect of block size and shape and efficiencies of different designs for bivariate, in general, multivariate situations.

When the variable to be analysed is a combined yield index such as LER, the univariate methods can be used directly as discussed in this paper.

From the C.V. values for the two approaches and from the curves in Figures 1 and 2, it is clear that for any chosen plot size, the precision attained through LER analysis is larger than through bivariate analysis.

In other words, per unit cost of experimentation will be larger, in order to attain the same level of precision, in the case of bivariate analysis. This is a result of considerable interest to agronomists and statisticians engaged in intercropping research, although this result has emerged as a by product of analysis of uniformity trial data.

## REFERENCES

[1] Hatheway, W. H. (1961) : Convenient plot size, *Agronomy Journal* **53** : 279-80.

[2] Mead, R. and Willey, R. W. (1980) : The concept of a 'Land Equivalent Ratio' and advantages in yields from intercropping, *Experimental Agriculture* **16** : 217-228.

[3] Narayana Reddy, M. and Ramanatha Chetty, C. K. (1982) : Effect of plot shape on variability in Smith's Variance law, *Experimental Agriculture* **18** : 333-38.

[4] Oyejola, B. A. and Mead, R. (1982) : Statistical assessment of different ways of calculating Land Equivalent Ratios, *Experimental Agriculture* **18** : 125-138,

[5] Pearce, S. C. and Gilliver, B. (1978) : The statistical analysis of data from intercropping experiments, *Journal of Agriculture Sciences*, Cambridge 91 : 625-32.

[6] Ramanatha Chetty, C. K. 1983) : Research on intercropping systems in Drylands. Project Bulletin No. 7. All India Co-ordinated Research Project on Dryland Agriculture, Hyderabad.

[7] Ramanatha Chetty, C. K. and Narayana Reddy, M. (1987) : Optimum size, shape and orientation of experimental plots with dryland sorghum, *Indian Journal of Agricultural Sciences* **51** (9) : 653-8.

[8] Rao, C. R. (1974) : *Linear Statistical Inference and its Applications*, Second Edition. pp 543-55, Wiley Eastern Private Ltd.

[9] Smith, H. F. (1938) : An empirical law describing heterogeneity in the yields of agricultural crops, *Journal of Agricultural Science*, Cambridge 28 : 1-23.

[10] Wilks, S. S. (1932) : Certain generalizations in the analysis of variance, *Biometrika* **24** : 471-494.